8. 
(a) For some positive integers $k$, the parabola with equation $y = \frac{x^2}{k} - 5$ intersects the circle with equation $x^2 + y^2 = 25$ at exactly three distinct points $A$, $B$ and $C$. Determine all such positive integers $k$ for which the area of $\triangle ABC$ is an integer.

16. If $x = 18$ is one of the solutions of the equation $x^2 + 12x + c = 0$, the other solution of this equation is

**(A)** $x = 216$ **(B)** $x = -6$ **(C)** $x = -30$ **(D)** $x = 30$ **(E)** $x = -540$

18. There are two values of $k$ for which the equation $x^2 + 2kx + 7k - 10 = 0$ has two equal real roots (that is, has exactly one solution for $x$). The sum of these values of $k$ is

**(A)** 0 **(B)** $-3$ **(C)** 3 **(D)** $-7$ **(E)** 7

3. 
(a) The line $y = -15$ intersects the parabola with equation $y = -x^2 + 2x$ at two points. What are the coordinates of these two points of intersection?


(b) A line intersects the parabola with equation $y = -x^2 - 3x$ at $x = 4$ and at $x = a$. This line intersects the $y$-axis at $(0, 8)$. Determine the value of $a$.


(c) A line intersects the parabola with equation $y = -x^2 + kx$ at $x = p$ and at $x = q$ with $p \neq q$. Determine the $y$-intercept of this line.


(d) For all $k \neq 0$, the curve $x = \frac{1}{k^3}y^2 + \frac{1}{k}y$ intersects the parabola with equation $y = -x^2 + kx$ at $(0, 0)$ and at a second point $T$ whose coordinates depend on $k$. All such points $T$ lie on a parabola. Determine the equation of this parabola.

22. If $x$ and $y$ are real numbers, the minimum possible value of the expression $(x+3)^2+2(y-2)^2+4(x-7)^2+(y+4)^2$ is

**(A)** 172 **(B)** 65 **(C)** 136 **(D)** 152 **(E)** 104

23. How many pairs $(x, y)$ of non-negative integers with $0 \leq x \leq y$ satisfy the equation $5x^2-4xy+2x+y^2=624$?

**(A)** 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

25. Points $P(r, s)$ and $Q(t, u)$ are on the parabola with equation $y=x^2-\frac{1}{5}mx+\frac{1}{5}n$ so that $PQ=13$ and the slope of $PQ$ is $\frac{12}{5}$. For how many pairs $(m, n)$ of positive integers with $n \leq 1000$ is $r+s+t+u=27$ ?

**(A)** 28 **(B)** 26 **(C)** 27 **(D)** 29 **(E)** 25

# Problems, May 2007

**Problem 1.** A circle meets the parabola $y=x^2$ at four points. The $x$-coordinates of three of the points are 2, 3, and 4. Find the $x$-coordinate of the fourth point.